Correspondence-Free Non-Rigid Point Set Registration Using Unsupervised Clustering Analysis

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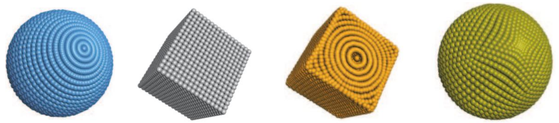
Project Page: (paper & code)

Motivation

- Registration without shape matching poses substantial challenges
- Unsupervised methods without annotations are more generalizable
- Iterative update of point positions is similar to clustering centroids update during unsupervised clustering process



Method

- Formulate non-rigid point set registration as clustering analysis
- Laplacian kernel function for robust displacement regularization
- Approximation error bound of the Nystrom low-rank method
- Dimension-independent, closed-form, robust to large deformations

The blue and gray models represent the source and target point clouds, respectively, while the yellow models are our registration results

- **problem formulation**

$$\mathcal{T}(\mathbf{Y}) \triangleq \mathbf{Y} + \nu(\mathbf{Y})$$

- **fuzzy clustering analysis**

$$\min F(\mathbf{U},\boldsymbol{\alpha},\boldsymbol{\Sigma},\nu) = \sum_{j=1}^{C}\sum_{i=1}^{M} u_{ij}||\boldsymbol{\Sigma}_j^{-\frac{1}{2}}(\boldsymbol{x}_i - (\boldsymbol{y}_j + \nu(\boldsymbol{y}_j)))||_2^2$$

$$+u_{ij}\log|\boldsymbol{\Sigma}_j| + \lambda u_{ij}\log\frac{u_{ij}}{\alpha_j},$$

$$s.t.\ |\boldsymbol{\Sigma}_j| = \theta_j, \sum_{j=1}^{C} u_{ij} = 1, \sum_{j=1}^{C}\alpha_j = 1, u_{ij}, \alpha_j \in [0,1].$$

- **deformation regularization**

$$\min F(\mathbf{U},\boldsymbol{\alpha},\boldsymbol{\Sigma},\nu) + \zeta\mathcal{R}(\nu)\ \ \mathcal{R}(\nu) = \int_{\mathbf{R}^n} d\mathbf{s}\frac{||\tilde{\nu}(\mathbf{s})||_2^2}{\tilde{K}(\mathbf{s})}$$

- **improved Nystrom approximation**

$$\epsilon \leq 4\sqrt{2}T^{3/2}\gamma\sqrt{C'q} + 2C'\gamma^2 Tq\|W^{-1}\|_F$$

- **closed-form solution**

$$\mathbf{U} = (\mathrm{diag}(\mathbf{A}\mathbf{1}_C))^{-1}\mathbf{A} \qquad \boldsymbol{\alpha} = \frac{1}{M}\mathbf{U}^T\mathbf{1}_M$$

$$\sigma^2 = \frac{\mathrm{tr}(\mathbf{X}^T\mathrm{diag}(\mathbf{U}^T\mathbf{1}_M)\mathbf{X} - (2(\mathbf{U}\mathbf{X})^T + \mathbf{T}^T\mathrm{diag}(\mathbf{U}\mathbf{1}_C))\mathbf{T})}{n \times M}$$

$$\mathbf{c} = (\mathbf{L} + \zeta\sigma^2\mathrm{diag}(\mathbf{U}\mathbf{1}_C)^{-1})^{-1}(\mathrm{diag}(\mathbf{U}\mathbf{1}_C)^{-1}\mathbf{U}\mathbf{X} - \mathbf{Y})$$

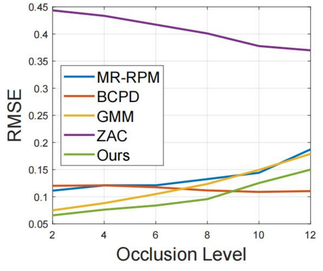
Result

- **2D hand pose registration**

Method	Subject 1	Subject 2	Subject 3	Subject 4	Time (s)
MR-RPM [30]	0.0940	0.0834	0.1028	0.1388	0.2382
BCPD [20]	0.1027	0.1055	0.1080	0.1579	0.6890
GMM [23]	0.0571	0.0547	0.0734	0.0917	0.1140
ZAC [45]	0.4886	0.4566	0.4879	0.4935	0.4254
Ours	**0.0383**	**0.0481**	**0.0537**	**0.0879**	**0.1074**

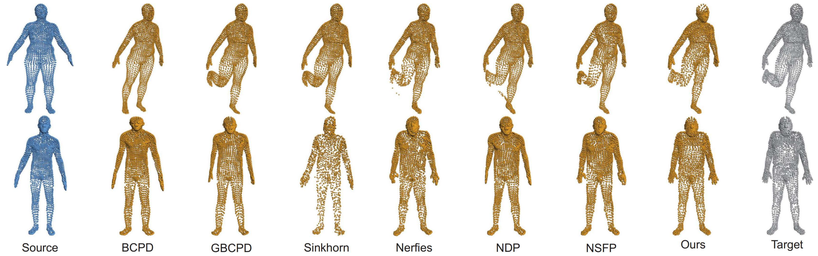
- **3D TOSCA dataset**

Method	Cat	Centaur	Dog	Gorilla	Average
BCPD [20]	3.9884	8.1017	7.2800	5.6253	5.9935
GBCPD [21]	1.5631	2.9480	1.5300	3.5751	2.6523
Nerfies [35]	3.2704	2.8826	1.3612	2.2809	2.3211
NDP [26]	4.3639	3.4373	3.1285	2.8312	3.2560
NSFP [24]	1.8774	2.6425	1.6734	2.2044	2.0710
Ours	**1.3496**	**1.8125**	**1.2088**	**1.6807**	**1.5247**

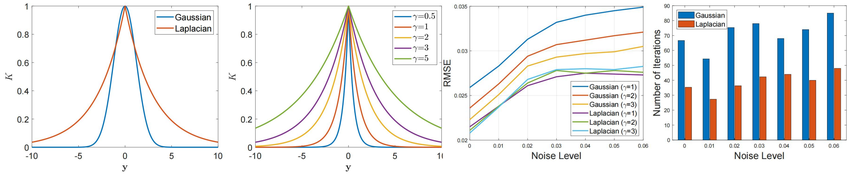


- **3D FAUST point cloud registration**

Method \ Settings	Intra-1	Intra-2	Intra-3	Intra-4	Intra-5	Intra-6	Inter-1	Inter-2	Inter-3	Inter-4	Average	Time (s)
BCPD [20]	0.0913	0.1011	0.0872	0.0577	0.1004	0.0746	0.1196	0.0705	0.0935	0.0923	0.0888	3.0359
GBCPD [21]	0.0285	0.0212	0.0211	0.0260	0.0244	0.0339	0.0359	0.0340	0.0212	0.0190	0.0265	1.9346
Fast_RNRR [47]	0.0430	0.0487	0.0397	0.0504	0.0429	0.0391	0.1358	0.0743	0.0477	0.0358	0.0557	**0.6324**
AMM_NRR [48]	0.0544	0.0486	0.0400	0.0539	0.0405	0.0393	0.0838	0.0686	0.0422	0.0399	0.0511	2.0438
Sinkhorn [16]	0.0654	0.0638	0.1372	0.1096	0.0749	0.0821	0.2467	0.0781	0.1400	0.1720	0.1170	2.0377
Nerfies [35]	0.0120	0.0107	0.0138	0.0129	0.0135	0.0118	0.0121	0.0144	0.0140	0.0140	0.0129	9.4287
NDP [26]	0.0183	0.0199	0.0192	0.0152	0.0170	0.0149	0.0181	0.0198	0.0164	0.0155	0.0174	1.7590
NSFP [24]	0.0126	0.0134	0.0132	0.0118	0.0137	0.0142	0.0167	0.0162	0.0148	0.0166	0.0143	2.4607
Ours	**0.0086**	**0.0089**	**0.0103**	**0.0096**	**0.0089**	**0.0081**	**0.0097**	**0.0099**	**0.0094**	**0.0081**	**0.0092**	2.3746



- **Ablation study**



- **Applications**