Accurate Registration of Cross-Modality Geometry via Consistent Clustering

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Joint work with

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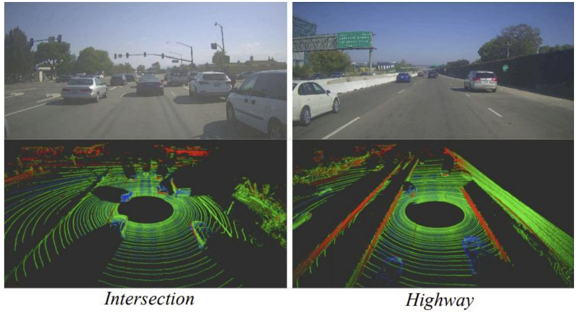
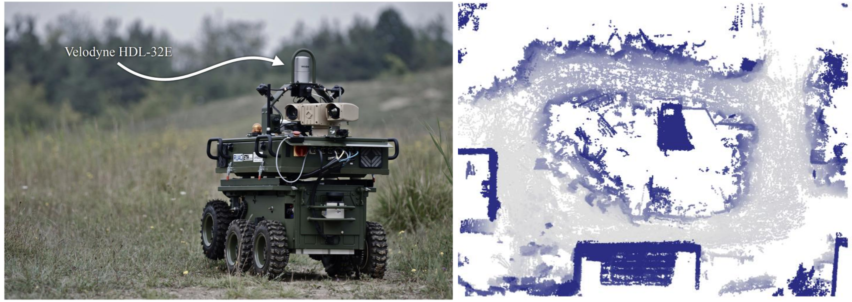
APPLICATIONS - SHAPE REGISTRATION

autonomous driving



[Lu et al. 2019]

robotics



[Pomerleau et al. 2015]

dynamic reconstruction

[Yao et al. 2024]

shape completion

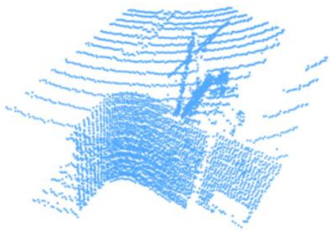
[Halimi et al. 2020]

MOTIVATION - CROSS-MODALITY REGISTRATION

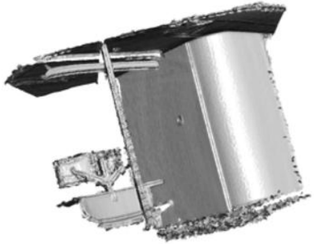
1. Cross-modality data having different representations, e.g., densities, textures, ...

2. Develop a holistic framework to deal with cross-modality registration

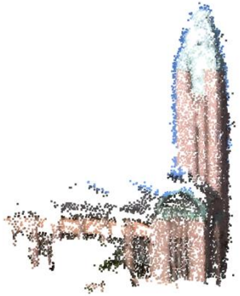
3. Consider three common modalities including LiDAR, Kinect, and MVS



LiDAR



Kinect

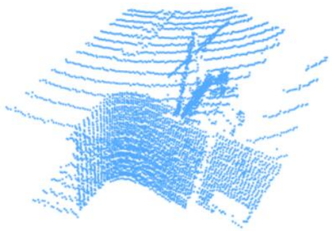


MVS

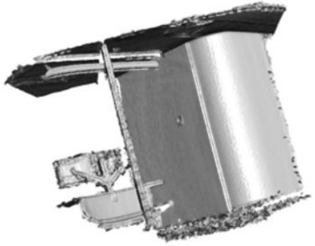
MOTIVATION - CROSS-MODALITY REGISTRATION

Observations: different representations BUT **similar geometric structures**

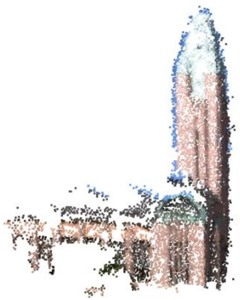
How to define geometric structures?



LiDAR



Kinect



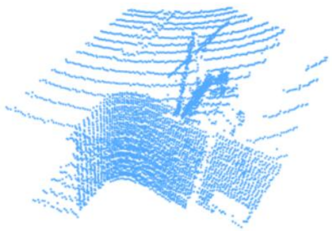
MVS

MOTIVATION - CROSS-MODALITY REGISTRATION

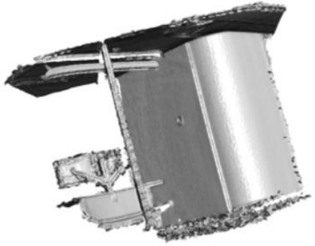
Observations: different representations BUT **similar geometric structures**

How to define geometric structures?

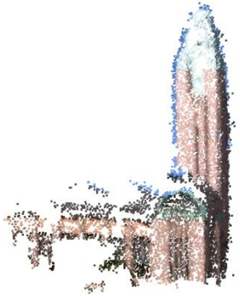
Clustering



LiDAR

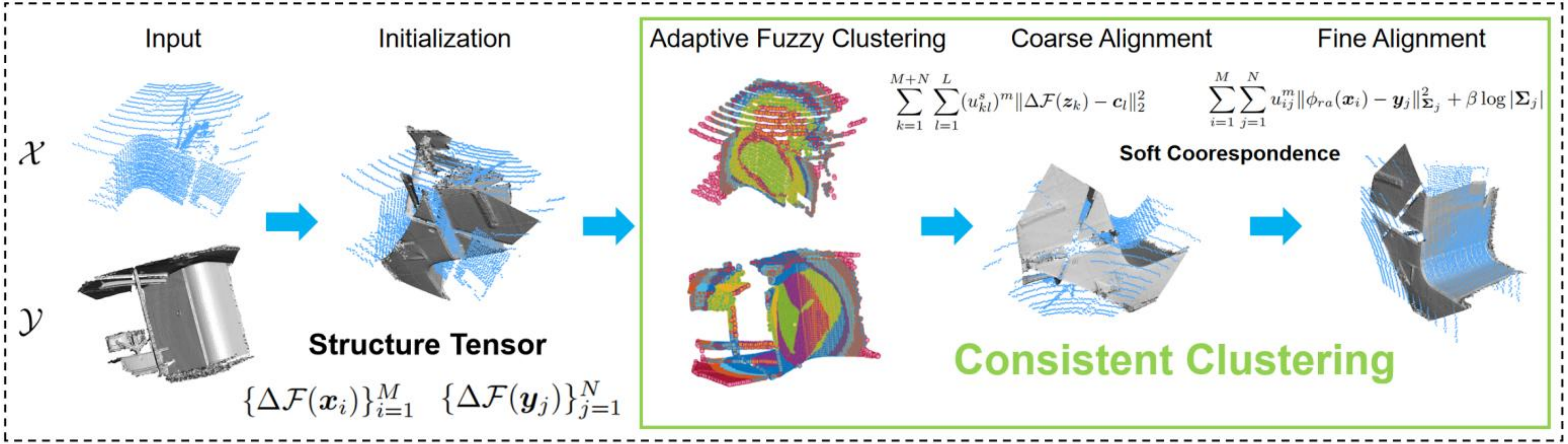


Kinect



MVS

METHOD - UNSUPERVISED CLUSTERING



Formulate cross-modality registration as **an unsupervised clustering problem**

METHOD - UNSUPERVISED CLUSTERING

Fuzzy clustering to discover similar structures

$$\boxed{\min_{\mathbf{U}_{\mathbf{s}},\mathcal{C}} J(\mathbf{U}_{\mathbf{s}},\mathcal{C}) = \sum_{k=1}^{M+N}\sum_{l=1}^{L}(u_{kl}^{s})^{m}\|\Delta\mathcal{F}(\boldsymbol{z}_k)-\boldsymbol{c}_l\|_2^2}$$

Clustering centroids

$$\boldsymbol{c}_l = \frac{\sum_{k=1}^{M+N} u_{kl}^{m}\Delta\mathcal{F}(\boldsymbol{z}_k)}{\sum_{k=1}^{M+N} u_{kl}^{m}}, \quad u_{kl}^{s} = \frac{\left(\frac{1}{\|\Delta\mathcal{F}(\boldsymbol{z}_k)-\boldsymbol{c}_l\|_2^2}\right)^{\frac{1}{m-1}}}{\sum_{l'=1}^{L}\left(\frac{1}{\|\Delta\mathcal{F}(\boldsymbol{z}_k)-\boldsymbol{c}_{l'}\|_2^2}\right)^{\frac{1}{m-1}}}$$

Fuzzy degree member

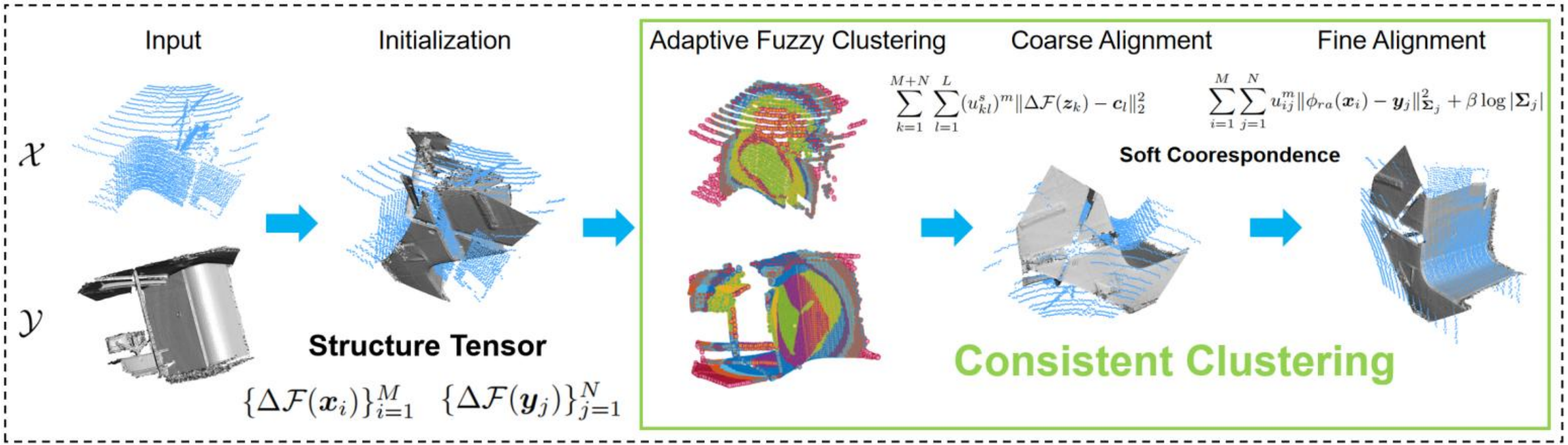
Moment difference

$$\Delta\mathcal{F}(\boldsymbol{x}) = (\Delta\mathcal{F}_1(\boldsymbol{x}), \Delta\mathcal{F}_2(\boldsymbol{x}), \cdots, \Delta\mathcal{F}_{n-1}(\boldsymbol{x})) \in \mathbb{R}^{n-1}$$

Multi-scale moment

$$\mathcal{F}_{r_i}(\boldsymbol{x}) = (I_{r_i1}, I_{r_i2}, I_{r_i3}) \quad I_1 = \frac{\lambda_3}{\lambda_1}, \quad I_2 = \sqrt{\lambda_1\cdot\lambda_2\cdot\lambda_3}, \quad I_3 = -\sum_{i=1}^{3}\lambda_i \ln \lambda_i$$

METHOD - COARSE REGISTRATION



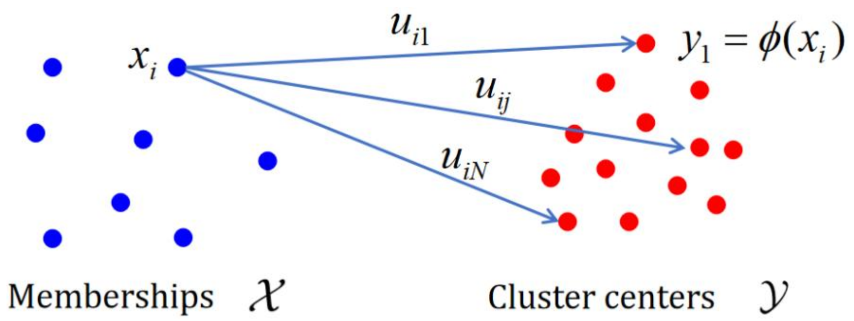
Coarse registration

$$\boxed{\phi_{ca} = \min_{\{\phi_l\}_{l=1}^{L}} \|\phi_l(\mathcal{X}) - \mathcal{Y}\|_2,\ s.t.\ \phi_l = \min_{\phi} \|\phi(\tilde{\mathcal{X}}) - \tilde{\mathcal{Y}}\|_2}$$

Fuzzy clustering to discover similar structures ⇨

$$\tilde{\mathcal{X}} = \{x_1, x_2, x_3\} \subset \mathcal{X},\ \tilde{\mathcal{Y}} = \{y_1, y_2, y_3\} \subset \mathcal{Y}$$

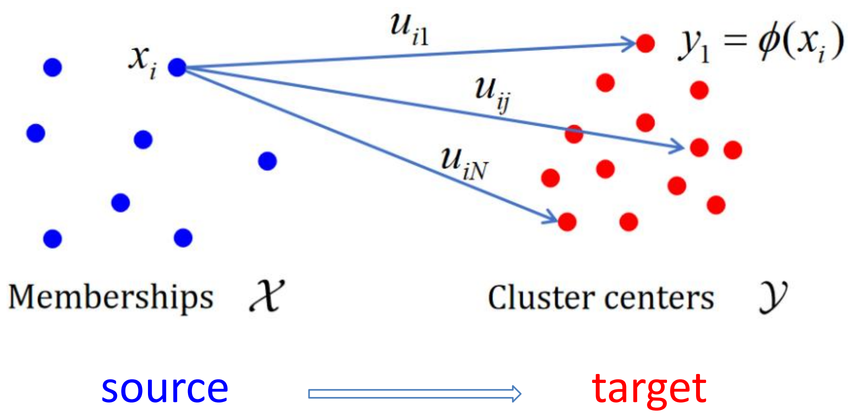
METHOD - FINE REGISTRATION



source $\Rightarrow$ target

Formulate fine point-wise alignment still as an unsupervised clustering process

METHOD - FINE REGISTRATION



$$\min_{\phi_{ra},\sigma^2} J(\phi_{ra},\sigma^2) = \sum_{i=1}^{M}\sum_{j=1}^{N} u_{ij}^{m} \|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|^2_{\boldsymbol{\Sigma}_j} + \beta \log|\boldsymbol{\Sigma}_j|$$

METHOD - FINE REGISTRATION

Objective function

$$\boxed{\min_{\phi_{ra},\sigma^2} J(\phi_{ra},\sigma^2) = \sum_{i=1}^{M}\sum_{j=1}^{N} u_{ij}^{m} \|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}^2 + \beta \log|\boldsymbol{\Sigma}_j|}$$

$$\phi_{ra}(\boldsymbol{x}_i) = \mathbf{R}\boldsymbol{x}_i + \mathbf{t}$$

$$\|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}^2 = (\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j)^T \boldsymbol{\Sigma}_j^{-1} (\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j)$$

Fuzzy degree member

$$u_{ij}^{d} = \frac{\left(\frac{1}{\|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}^2}\right)^{\frac{1}{m-1}}}{\sum_{k=1}^{N}\left(\frac{1}{\|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_k\|_{\boldsymbol{\Sigma}_k}^2}\right)^{\frac{1}{m-1}}} = \frac{1}{\sum_{k=1}^{N}\left(\frac{\|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}}{\|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_k\|_{\boldsymbol{\Sigma}_k}}\right)^{\frac{2}{m-1}}}$$

METHOD - FINE REGISTRATION

Objective function

$$\boxed{\min_{\phi_{ra},\sigma^2} J(\phi_{ra},\sigma^2) = \sum_{i=1}^{M}\sum_{j=1}^{N} u_{ij}^{m} \|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}^2 + \beta \log |\boldsymbol{\Sigma}_j|}$$

Theorem 2. *The upper bound of the fuzzy registration residual (Eq. 4) is minimized by the following equations:*

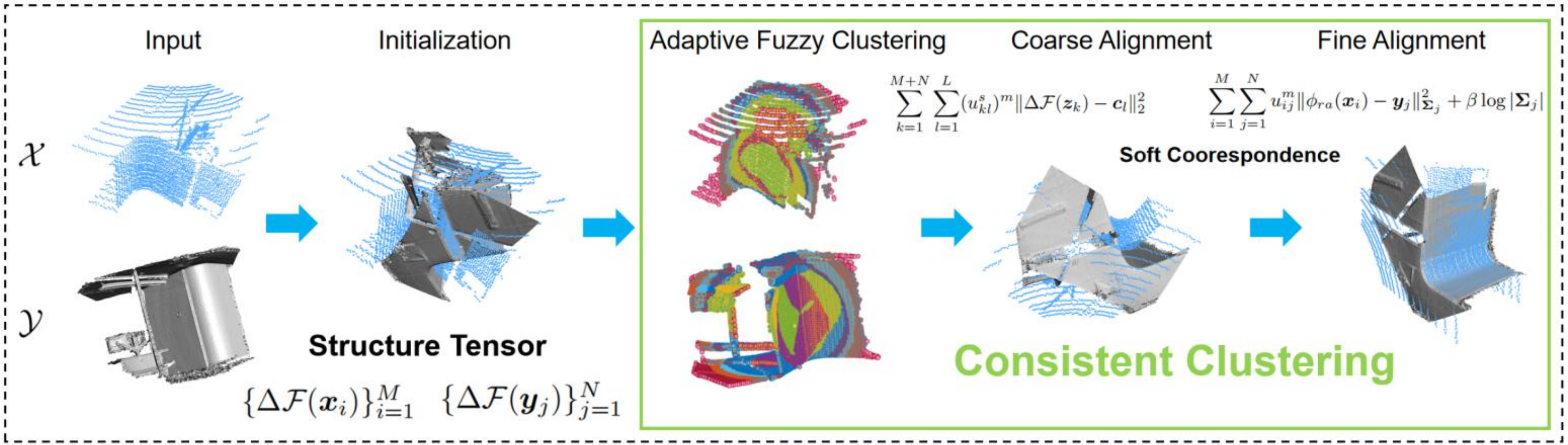
$$\hat{\mathbf{R}} = \boldsymbol{\Phi}\mathbf{d}(1,\cdots,1,|\boldsymbol{\Phi}\boldsymbol{\Psi}^T|)\boldsymbol{\Psi}^T, \quad \hat{\mathbf{t}} = \bar{\boldsymbol{y}} - \hat{\mathbf{R}}\bar{\boldsymbol{x}},$$

$$\hat{\sigma}^2 = \frac{1}{3S}\mathrm{tr}(\bar{\mathbf{X}}\mathbf{d}(\mathbf{U}\mathbf{1})\bar{\mathbf{X}}^{\mathbf{T}} - \mathbf{2}\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Psi}\hat{\mathbf{R}}^{\mathbf{T}} + \bar{\mathbf{Y}}\mathbf{d}(\mathbf{U}^{\mathbf{T}}\mathbf{1})\bar{\mathbf{Y}}^{\mathbf{T}}),$$

where $\mathbf{X} = [\boldsymbol{x}_1,\cdots,\boldsymbol{x}_m] \in \mathbb{R}^{3\times M}$, $\mathbf{Y} = [\boldsymbol{y}_1,\cdots,\boldsymbol{y}_n] \in \mathbb{R}^{3\times N}$, $S = \sum_{i,j=1}^{M,N} u_{ij}$, $\bar{\mathbf{x}} = \frac{1}{S}\mathbf{X}\mathbf{U}\mathbf{1}$, $\bar{\boldsymbol{y}} = \frac{1}{S}\mathbf{Y}\mathbf{U}^{\mathbf{T}}\mathbf{1}$, $\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\mathbf{T}} = svd((\mathbf{Y}-\bar{\boldsymbol{y}}\mathbf{1})\mathbf{U}^{\mathbf{T}}(\mathbf{X}-\bar{\boldsymbol{x}}\mathbf{1})^{\mathbf{T}}) = svd(\bar{\mathbf{Y}}\mathbf{U}^{\mathbf{T}}\bar{\mathbf{X}}^{\mathbf{T}})$, $\mathbf{1}$ *is the corresponding full one column vector.*

Analytical closed-form solution (all unknowns) ⇒ Fast alternative optimization (coordinate descent)

METHOD – CROSS-MODALITY REGISTRATION



discover structure similarity | coarse | fine

Formulate cross-modality registration as **an unsupervised clustering problem**

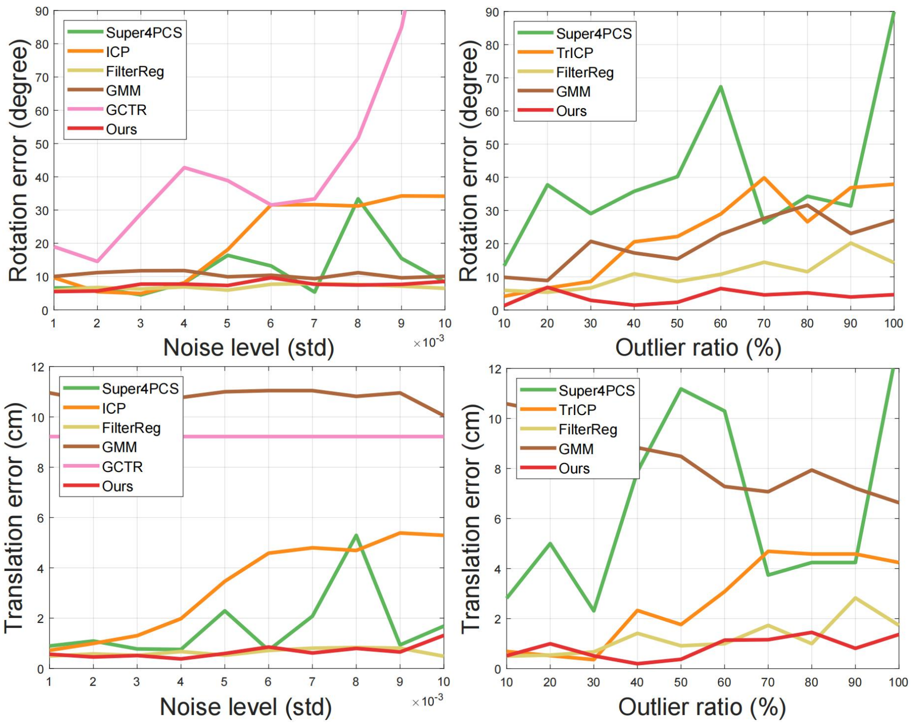
METHOD - RELATION WITH ICP

Lemma 1. *Let* $I = \{1, \cdots, z\} \subseteq Z^+$ *is an index set,* $a_{i \in I} \in \mathbb{R}^+$ *and* $p \in (0, +\infty)$*, then*

$$z^{-\frac{1}{p}} \min_{i \in I} a_i \leq \left(\sum_{i=1}^{z} a_i^{-p}\right)^{-\frac{1}{p}} \leq \min_{i \in I} a_i .$$

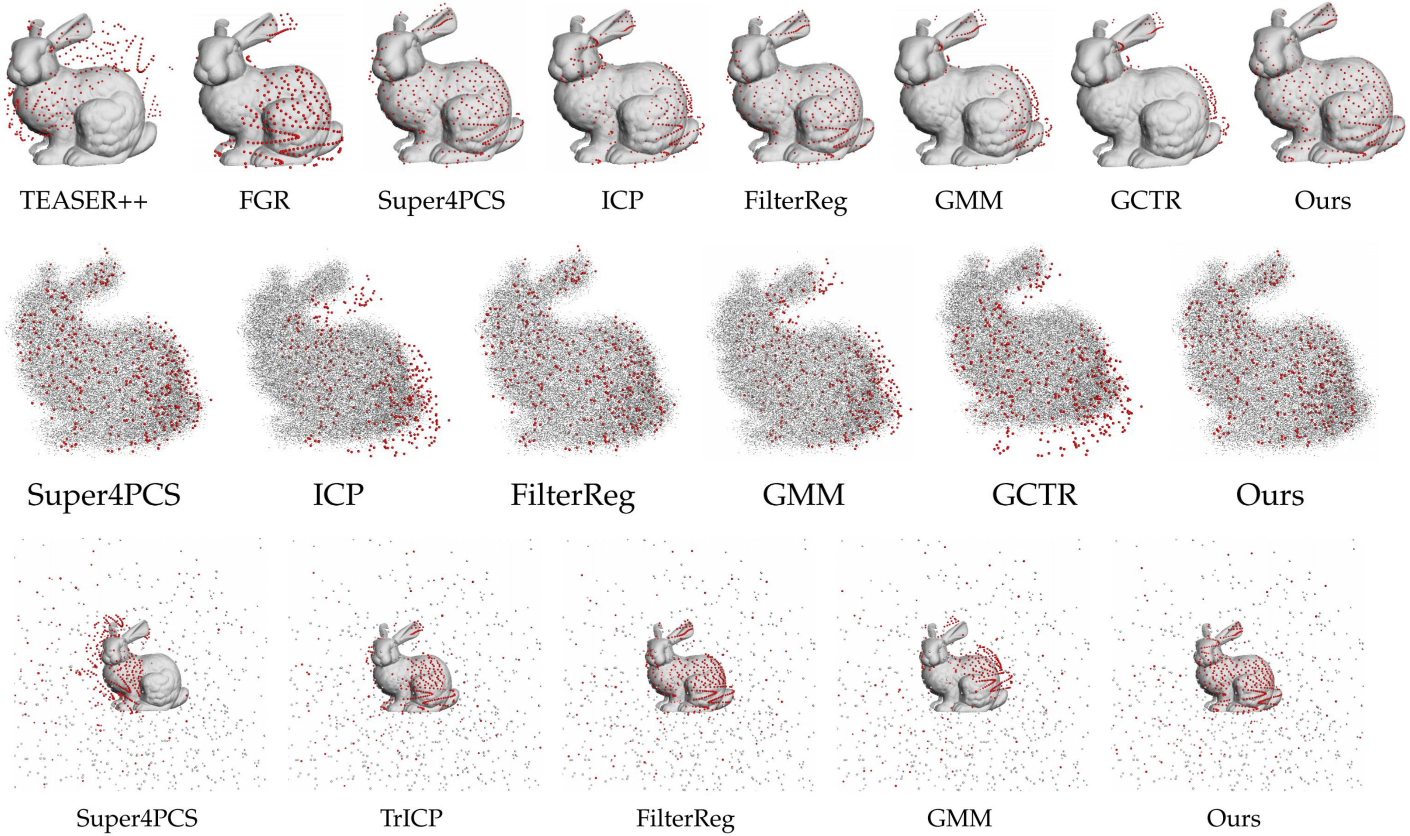
Theorem 1. *The iterative closest point (ICP) algorithm is a special case of the proposed fuzzy registration method when the fuzzier* $m \in (1, +\infty)$ *converges to 1 and* $\mathbf{\Sigma} = \mathbf{I}$.

RESULTS - SYNTHETIC TEST



noise & outlier test

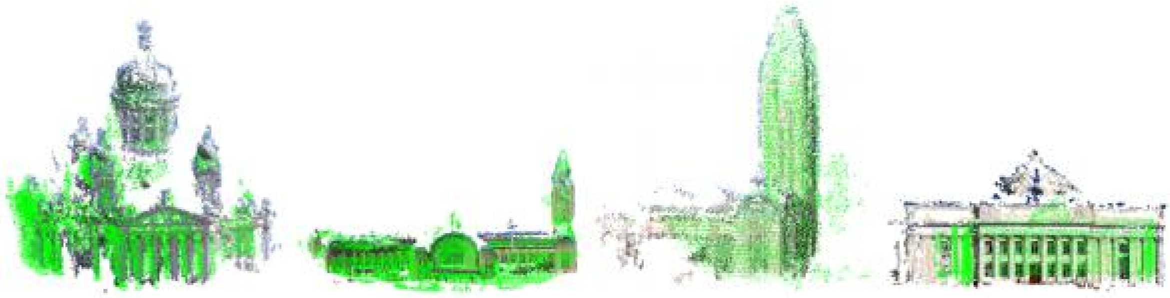
RESULTS - SYNTHETIC TEST



RESULTS - LiDAR & MVS

Real-world Helsinki outdoor dataset [Peng et al. 2014]

Dataset	Metric	Super4PC [22]	TrICP [17]	FilterReg [29]	GMM [26]	GCTR [9]	Ours
Library	RE	165.1815	84.7910	60.2347	4.8023	163.2037	**3.2664**
	TE	0.4297	0.1828	0.2316	0.2039	0.4136	**0.1063**
Cathedral	RE	79.9109	11.6182	**2.3517**	11.8433	121.6016	6.0415
	TE	1.0391	0.2972	**0.1154**	1.4506	1.0503	0.2782
East Station	RE	85.4812	15.2125	1.4468	2.5072	77.6636	**0.9823**
	TE	1.7305	0.1519	0.0409	0.3290	0.2513	**0.0341**
South Station	RE	147.1696	1.9297	0.9860	4.9186	106.8087	**0.8726**
	TE	0.0921	**0.0015**	0.0347	0.0563	0.0489	0.0456



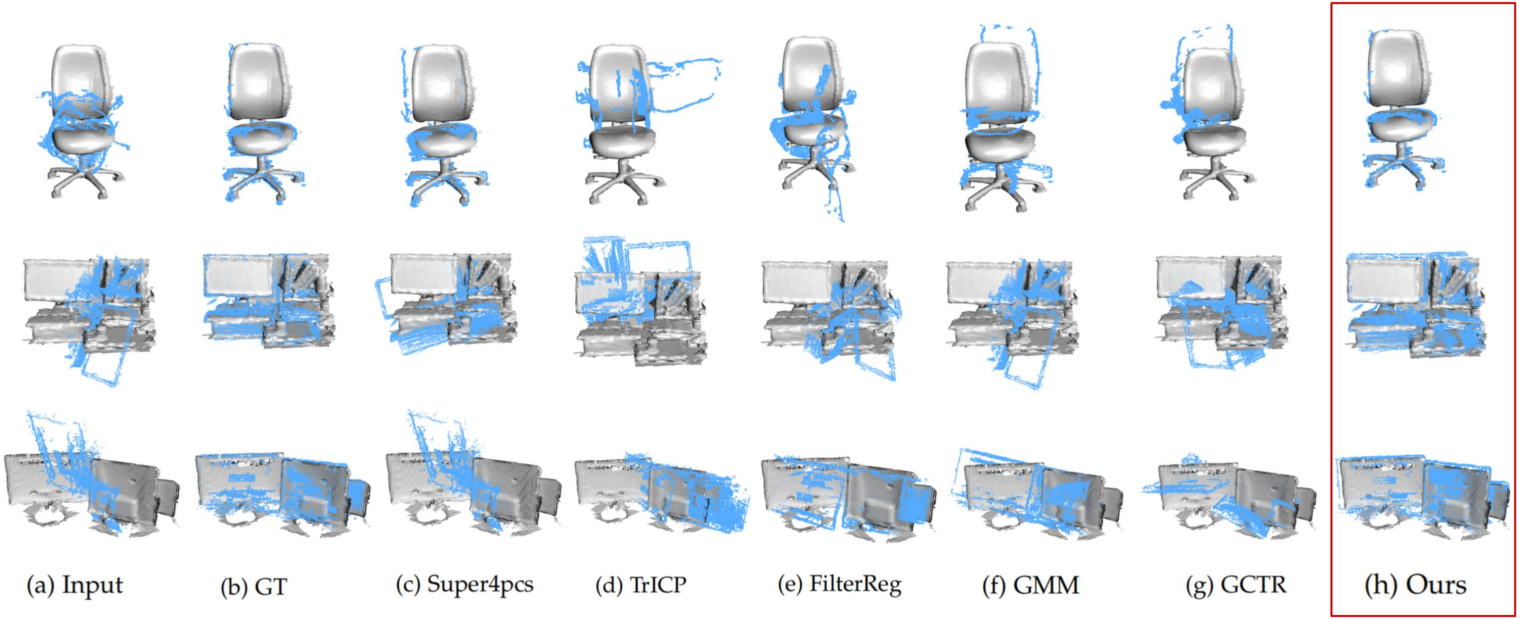
Align real-world LiDAR data to MVS data

RESULTS - LiDAR & KINECT

Real-world 3DCSR indoor dataset [Huang et al. 2019]

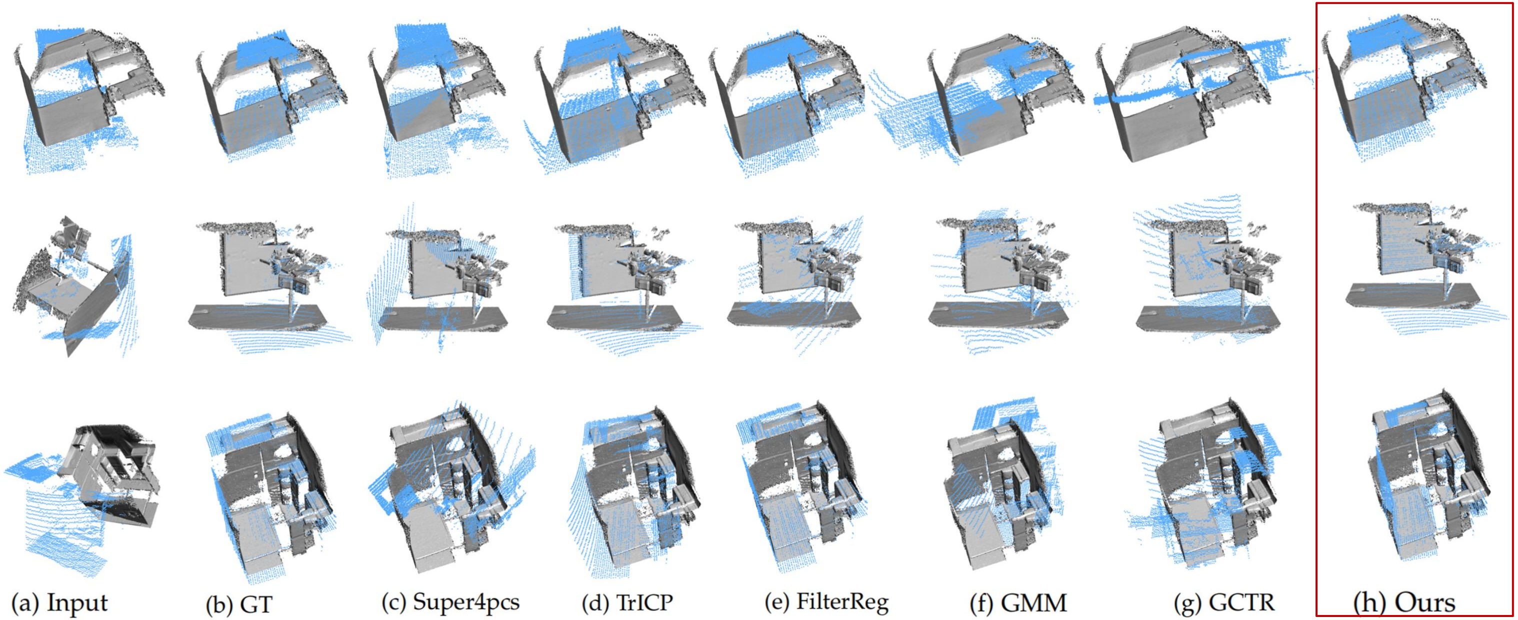
Method	Recall↑	TE↓	RE↓	Time(s)↓
DGR [39]	36.60	**0.04**	4.26	0.87
FMR [58]	17.80	0.10	4.66	0.28
PointNetLK [37]	0.05	0.09	12.54	2.25
FGR [19]	1.49	0.07	10.74	2.23
ESF-64+ICP [43]	24.30	**0.04**	5.71	0.19
JRMPC [59]	0.00	-	-	18.10
RANSAC [10]	3.47	0.13	8.30	**0.03**
GCTR [9]	0.50	0.17	7.46	15.80
Super4PCS [23]	6.93	0.24	6.38	1.70
TrICP [18]	7.92	0.18	6.40	1.26
FilterReg [32]	30.96	0.10	2.45	7.06
GMM [29]	9.41	0.18	7.92	13.34
CICP [45]	2.48	0.28	8.28	0.61
PICP [26]	4.45	0.29	10.85	12.58
Ours	**40.59**	0.06	**2.21**	4.47

RESULTS - LiDAR & KINECT



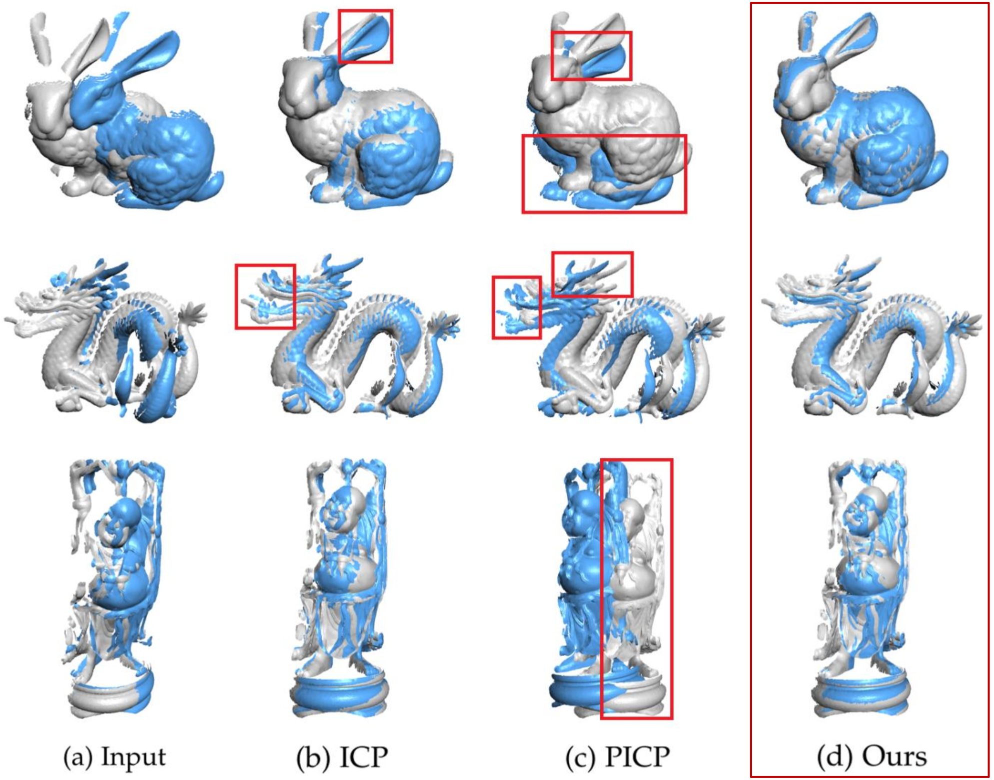
Real-world 3DCSR indoor dataset [Huang et al. 2019]

RESULTS - LiDAR & KINECT



Real-world 3DCSR indoor dataset [Huang et al. 2019]

RESULTS - SAME MODALITY

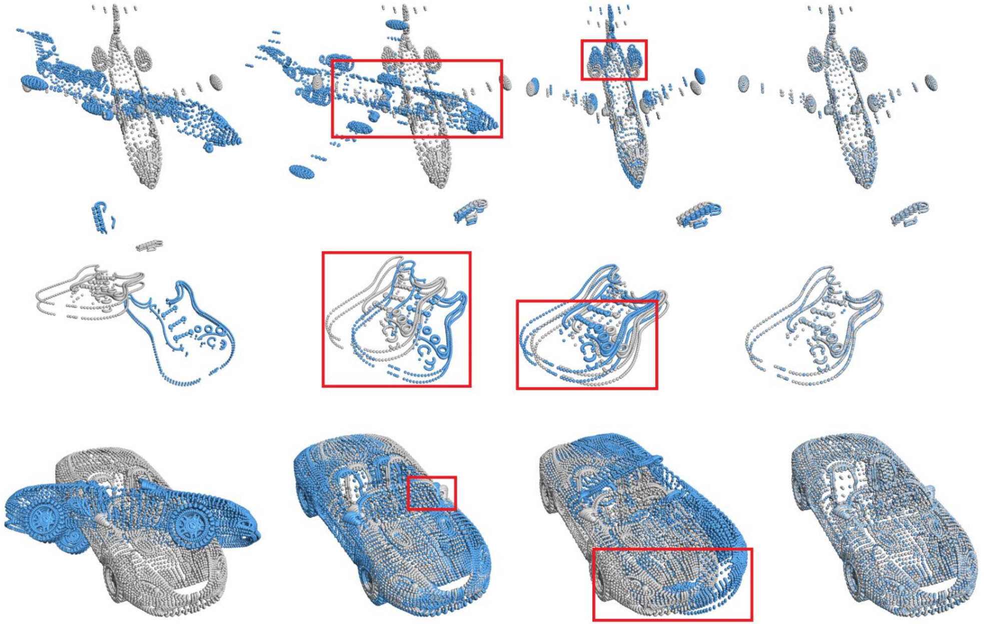


Dataset / Method	Bunny		Dragon		Happy Buddaha	
	RE↓	TE↓	RE↓	TE↓	RE↓	TE↓
ICP [13]	4.2717	0.0022	4.6766	**0.0060**	5.0297	**0.0016**
PICP [26]	21.5870	0.1286	17.7415	0.0184	25.7583	0.0154
Ours	**1.2439**	**0.0013**	**1.3350**	0.0104	**0.4647**	0.0020

Dataset / Method	Airplane		Guitar		Car	
	RE↓	TE↓	RE↓	TE↓	RE↓	TE↓
ICP [13]	54.3791	0.5157	19.2938	0.1412	5.5450	0.0762
PICP [26]	10.8500	0.0844	5.2585	0.0429	193.3783	0.2240
Ours	**1.0001e-6**	**1.5128e-7**	**2.3286e-6**	**9.5704e-6**	**2.3454e-6**	**8.9725e-8**

The Stanford 3D scanning repository

RESULTS - SAME MODALITY

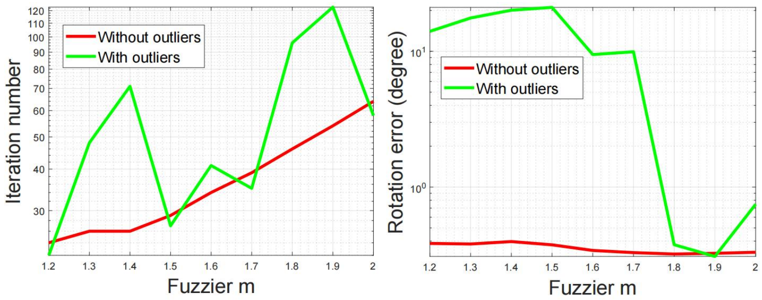


(a) Input (b) ICP (c) PICP (d) Ours

ModelNet 40

RESULTS - INVESTIGATION OF THE FUZZIER

$$\min_{\phi_{ra},\sigma^2} J(\phi_{ra},\sigma^2) = \sum_{i=1}^{M}\sum_{j=1}^{N} u_{ij}^m \|\phi_{ra}(\boldsymbol{x}_i) - \boldsymbol{y}_j\|_{\boldsymbol{\Sigma}_j}^2 + \beta \log|\boldsymbol{\Sigma}_j|$$



1. As m increases, iterations increase, rotation error decreases

2. Recommend $m \in [1.8, 2]$ in use

LIMITATIONS & FUTURE WORK

- Boost the efficiency with parallel optimization
- Compile more cross source data
- Extend to non-rigid scenarios using unsupervised clustering

CONCLUSION

- Take structure similarity handling cross-modality registration
- Formulate cross-modality registration (coarse&fine) as a clustering process
- Demonstrate impressive performance on common data modalities

THANK YOU FOR YOUR ATTENTION!